NUMERICAL SIMULATION OF A RELATIVISTIC ELECTRON BEAM IN A DRIFT TUBE WITH FINITE EXTERNAL MAGNETIC FIELD

N. I. Sablin, T. A. Solod,
and G. P. Fomenko

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The "current-tube" method is used to solve the problem of numerical simulation of a steady-state axially symmetrical relativistic electron beam in a drift tube with finite external magnetic field. The geometry of the problem and an algorithm for solution were presented in [1]. With the assumption of an infinite external longitudinal magnetic field [1] simulated a relativistic electron beam, for which case purely longitudinal motion and insignificance of the effect of the intrinsic magnetic field $\mathbf{B}^i$ are characteristic. In a finite external magnetic field $\mathbf{B}^i$ beam motion is determined by three velocity components and consideration of the effect of all components of $\mathbf{B}^e$ on the beam is required. Moreover, the relationships between the components of $\mathbf{B}^i$ and the character of their distributions is of importance in analyzing the external beam structure.

The mathematical model consists of the system of Maxwell's equations and the equations of motion of the relativistic electrons in a vacuum:

$$\nabla^2\varphi = -4\pi\rho, \quad E_r = -\frac{\partial\varphi}{\partial r}, \quad E_z = -\frac{\partial\varphi}{\partial z}, \quad \nabla^2 A_\theta - \frac{A_\theta}{r^2} = -\frac{4\pi}{c} j_\theta,$$
$$B_r^i = -\frac{\partial A_\theta}{\partial z}, \frac{1}{r}\frac{\partial}{\partial r} rB_\theta^i = \frac{4\pi}{c} j_z, \quad B_z^i = \frac{1}{r}\frac{\partial}{\partial r} rA_\theta, \tag{1}$$
$$\frac{d\mathbf{p}}{dt} = \frac{e}{m_0}\mathbf{E} + \frac{e}{m_0 c}\frac{[\mathbf{p}, \mathbf{B}]}{(1+p^2/c^2)^{1/2}}, \quad \mathbf{p} = \mathbf{v}/(1-v^2/c^2)^{1/2}$$

with zero boundary conditions

$$\varphi, A_\theta|_{z=0,L} = 0; \ \varphi, A_\theta|_{r=R} = 0; \ \frac{\partial\varphi}{\partial r}, \frac{\partial A_\theta}{\partial r}\bigg|_{r=0} = 0, \tag{2}$$

where $\mathbf{B} = \mathbf{B}^e + \mathbf{B}^i$; $\mathbf{B}^e = (0, 0, B_z^e)$; $\mathbf{B}^i = (B_r^i, B_\theta^i, B_z^i)$.

The values of current density and space charge required to complete system (1), (2) are calculated by the "current-tube" method. The fact that Eq. (1) does not explicitly contain the radial current component j_r is a consequence of the axial symmetry of the problem and the method chosen for its solution (the "current-tube" method). A solution constructed in this manner will automatically satisfy the equation $\operatorname{div}_j = 0$, and this relationship was used to monitor the correctness of the calculation.

In contrast to [1], the charge q_k^t is concentrated in a volume V_k^t, limited by cylindrical surfaces with radii $r = \min\{r_{k-1}^t, r_{k-1}^{t+1}\}$, $r = \max\{r_{k+1}^t, r_{k+1}^{t+1}\}$ and planes $z = z_k^t$, $z = z_k^{t+1}$.

The components of the current density j_θ and j_z are calculated with the expression $j_{xij} = \rho_{ij} v_{xij}$, where $v_{xij} = \sum V_{kij}^t (v_{xk}^t + v_{xk}^{t+1})/2 \sum V_{kij}^t$; $x = \theta, z$; v_{xij} is the corresponding component of the velocity v_{ij}.

As in [1], for solution of system (1), (2), we use the method of space charge relaxation in the volume

$$\Delta_h \varphi^{s+1} = -4\pi\rho^s, \ \rho^{s+1} = \omega_s \hat{\rho}^{s+1} + (1-\omega_s)\rho^s, \ s = 0, 1, 2, \ldots, \tag{3}$$

where Δ_h is a difference analog of the Laplace operator; ω_s is the sequence of relaxation parameters; $\hat{\rho}^{s+1}$ is the density of the space charge reconstructed in each cell of the grid $\bar{\omega}_{hl}$ by the method of "smearing" over areas for $\varphi = \varphi^{s+1}$. To accelerate the convergence of the iteration process and so that for input currents I_+ greater than a limiting value the beam will more rapidly take on the well-known "glass" form, in the first approximation we take $\omega_s = 0.3$, after which the value of ω_s is decreased rapidly to 0.01.

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Calculations were performed for various values of input current I_+ and B_z^e, with a drift tube radius R = 4.6 cm, and an electron beam injected with uniform current density and initial radius r_b = 2 cm: For sublimiting current values for the given geometry, I_+ = 7 kA and B_z^e = 3 kG, the iteration process converges in six iterations for $\omega_s = 1$. For an input current above the limiting value (I_+ = 20 kA) the problem was solved for B_z^e = 3.5 kG, tube lengths of 20 and 30 cm, and input beam kinetic energy W_+ = 1 MeV. At B_z^e = 5 kG, L = 20 cm the iteration process Eq. (3) is established at 27 iterations, after which the current at the drift tube output oscillates within the limits 6.7 kA $\leqslant I_- \leqslant$ 7.1 kA. With decrease in external magnetic field (B_z^e = 3 kG) the instability of the problem intensifies, and at L = 30 cm iteration process (3) does not converge, and large oscillations in output current I_- occur. At L = 20 cm the iteration process also does not converge, although the amplitude of I_- oscillations decreases, which may be explained by an insignificant increase in limit current.

With the given input parameters one of the causes of instability of the iteration process (3) is the monoenergetic nature of the input beam, which leads to the appearance of singularities in the function $\rho(r, z)$ in the virtual cathode region. In connection with this, even insignificant changes in density ρ^s cause large oscillations in I_-, i.e., the problem becomes unstable (incorrect, by Tikhonov's definition — the third correctness condition is violated [2]). The singularity in the function $\rho(r, z)$ can obviously be smoothed if we specify an I_+ distribution function over longitudinal or transverse velocities.

For example, let a distribution function over longitudinal velocity be specified. We divide I_+ into n energy groups such that $I_+ = \sum_{i=1}^{n} \alpha_i I_+$, where α_i are weights determined by the energy distribution $\left(\sum_{i=1}^{n} \alpha_i = 1\right)$. Now at each point of the plane z = 0, which is the initial coordinate of the current tube, there will be emitted not one, but n tubes with current $\alpha_i I_+^h$ (I_+^h is the current in tube k) and with $\beta_i = v_{zi}/c$, corresponding to energy group i. In this case the virtual cathode region expands and the gradient of the function $\rho(r, z)$ decreases, causing a reduction in the order of its singularity.

To produce a more realistic approximation of the beam structure and to illustrate the above points, an electron beam was modeled for B_z^e = 3 kG, L = 30 cm with a close to real energy spread, not exceeding 10% of W_+, with the following parameters: $\alpha_1 = 0.03$, $\alpha_2 = 0.15$, $\alpha_3 = 0.32$, $\alpha_4 = 0.5$, $\beta_1 = 0.914$, $\beta_2 = 0.927$, $\beta_3 = 0.942$, $\beta_4 = 0.945$, v_r, $v_\theta = 0$. As an initial approximation for iteration process (3) the results of a calculation without consideration of the I_+ distribution over longitudinal velocity were chosen. Convergence of Eq. (3) was then achieved in 15 iterations, with an output current $I_- \approx 8.5$ kA.

Results of solving Eqs. (1), (2) for an input current I_+ = 20 kA are presented in Figs. 1-5. Figure 1 shows a graph of φ for B_z^e = 3 kG (solid lines) and 5 kG (dashed lines). For B_z^e = 3 kG Figs. 2-4 show graphs of B_r^i, B_θ^i, B_z^i. It is evident from Figs. 1-4 that the quantity φ and the B_z^i components reach extremal values in the virtual cathode region, with the highest value $B_\theta^i \approx 1.4$ kG, $B_z^c \approx 0.35$ kG. The character of the change in intrinsic magnetic components shows that the effect of B^i on the relativistic electron beam is most significant in the virtual cathode region, while beyond the virtual cathode its effect is insignificant in practice. The azimuthal component of the intrinsic magnetic field B_θ^i proves to have a

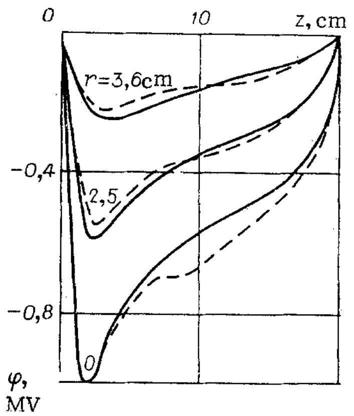

Fig. 1

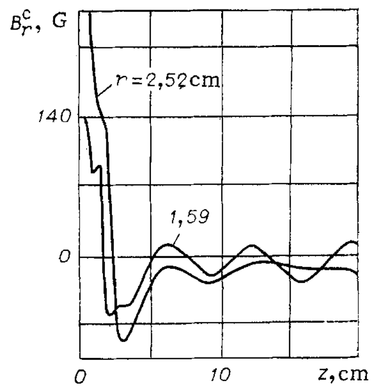

Fig. 2

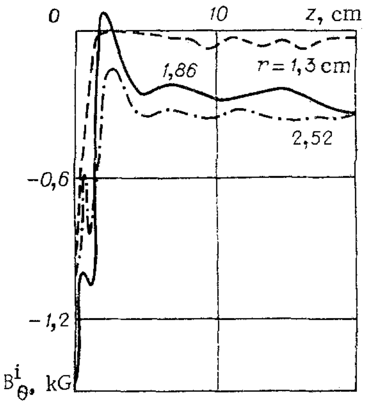

Fig. 3

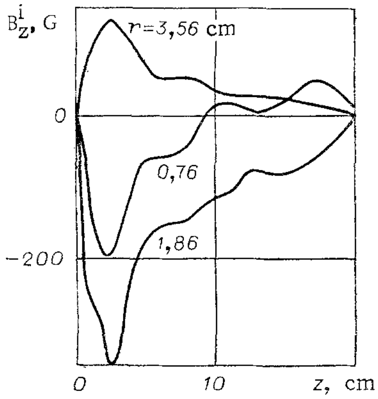

Fig. 4

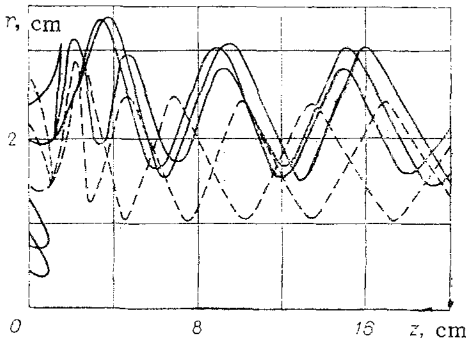

Fig. 5

significant focusing effect on the beam, especially at the entrance to the drift space, where $E_r \approx 0$. This is shown in Fig. 5, where characteristic trajectories of "current tubes" are shown: Near the injection plane the trajectories are directed toward the axis of symmetry. The longitudinal component B^i_z, has a sign opposite that of B^e_z, which increases the cyclotron radius of the electrons. Figure 5 also shows that due to formation of a virtual cathode a portion of the trajectories return to the plane z = 0. The mean values of output current I_- obtained by calculation agree with the experimental measurements of [3].

It follows from the calculations performed that in numerical simulation of a steady-state relativistic electron beam at input currents above the limit value in metallic drift tubes at $B^e_z \geqslant 5$ kG calculations with monoenergetic input beams can be recommended, while in the opposite case to insure stability of the iteration process (3) it is necessary to use an energetic or angular distribution of the input current.

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